Study Linear Algebra at MIT with Gilbert Strang

Posted on 15 April 2011 by **cjf**

Glibert Strang's 18.06 Linear Algebra course at MIT OpenCourseWare is exquisite! Jeannie and I went through it about a year ago. Strang's approach to the material and engaging teaching style make the course a joy. Unlike other OER (Open Educational Resources) courses that we have taken, I cannot recommend just watching the videos. Instead one needs to really think about the concepts which is greatly facilitated by doing the exercises: this is typical for mathematics. It took significant effort to master the content. The material builds quickly and it was essential for us to work hard on each lecture. On several occasions we re-watched the videos and had to read the text carefully and collate with other on-line resources (some provided or referenced in the wonderful OpenCourseWare materials but some found by web searching, Wikipedia, etc.). We also used resources from our library (personal and public).

Jeannie and I were able to enjoy the whole course but it definitely took significant effort. This is the only video course that we have thoroughly studied and it was worth it!

Are there any other OER video courses on linear algebra? I have not found any which is a shame for such important material.

Linear algebra is one of the most useful branches of mathematics beyond introductory (high school) algebra and geometry. It is the algebraic study of intersections of complexes of lines, planes and hyperplanes and therefore has a strong geometrical component. Since each coordinate in the Cartesian representation of a line can be thought of as a variable, linear algebra provides a first order or "linear" approximation to multivariable systems. It is therefore a widespread and fundamental tool. Linear algebra has found applications in business, economics, engineering, genetics, computer graphics, social sciences, graph theory and much more. It is essential for anyone wanting to understand advanced mathematics. The matrix or an array of numbers is the basic object of study in linear algebra. So it is sometimes called matrix theory. Vector spaces are the abstract form of linear algebra.

Is that a good characterization of linear algebra? How would you improve it?

The calculus prerequisite is used at several points, but those could be skimmed or skipped as they are not essential to the development. The only critical prerequisites for the course are competency with high school algebra and arithmetic (logarithms and trigonometry show up but are not essential) and a good understanding of basic analytic geometry.

In 1986 at Binghamton University I studied Linear Algebra with Fenando Guzman which was an excellent experience that I still remember fondly. That course approached the material in a more traditional "abstract" style. Strang's concrete approach with proofs that rarely go beyond one or two lines was refreshingly accessible and delightfully "clean" … exposing the elegance of the subject in a way that proof-oriented courses sometimes obscure. With proofs less "in your face", Strang can concentrate on motivation and seeing the "big picture" … the connections between the ideas without getting bogged down in the pedantics of logic or the peculiarities of the stilted style of modern mathematical proofs. I value proofs but I find that in most of the math that I read (and I continue to read mathematics fairly extensively) the proofs fail to provide the kind of intuitive grasp of the material that is necessary for a deeper understanding. They provide an abbreviated summary that is less nourishing (an often less convincing) than the kind of mathematics that Strang thrilled me with in his course.

We got Strang's book out on interlibrary loan, worked many of the exercises (more than were assigned to the class), worked through all three quizzes, and completed two and a half final exams. Strang's book is pretty essential for the course. We used an older edition and that was not too bad, but from what I can tell from a preview at Google Books the newer editions would have made life easier. I found six alternative OER (Open Educational Resources) linear algebra texts cited by Wikipedia. I can recommend Jim Hefferon's excellent book but its style and content is so different from Strang's approach that it wasn't very useful for the course. Robert A. Beezer's book is also a good supplement but I found it to be too tedious. I did not have time to carefully review the works by Keith Matthews, Sergei Treil, or Kardi Teknomo, but they look promising. The books by Edwin H. Connell and Rusian Shapirov are too abstract for one's first exposure to the subject. We used some of the Khan Academy videos on linear algebra but they are so short that I only found them useful to help with topics on which I wanted to see another presentation for extra perspective.

It took a sustained and concerted effort to get through the course. Especially since interlibrary loan, regrettably, does not permit a sufficient number of renewals. But that fact made us work hard to finish it before the book had to be returned! I would not have passed the finals without the "open book" approach that we permitted ourselves (we were interested in learning the material and do not need to keep it in the grey cells at all times: indeed we have already "lost" much of the content through atrophy).

I highly recommend this course!

There were several features of Stang's course that stood out to me as worthy of note:

- Matrix focus (that is, more concrete and less abstract)
  - Thinking of a matrix as an "action" throughout
  - Column vector focus (row vectors are x transpose represented as x')
  - Slicing and dicing matrix multiplication in five different ways:
    - Traditional row by column
    - column by row picture: AB $=a_1b_1 + \dots + a_nb_n$ where a_i is a column of A and b_i is a ROW of B (so you get the sum of n rank one matrices)
    - Combinations of columns of A picture (multiplication on the left acts on columns)
    - Combinations of rows of B picture (multiplication on the right acts on rows)
    - Traditional block multiplication
- An interesting take on "the fundamental theorem of linear algebra":
  - Characterization of the four fundamental subspaces: The Column Space C(A), the Nullspace N(A), the Row Space C(A'), and the "left nullspace" N(A') where A' means A transpose
  - C(A) and C(A') have dimension r = rank of A = # of pivots
  - N(A) has dimension n-r; N(A') has dimension m-r
  - C(A') is the orthogonal complement of N(A)
  - C(A) is the orthogonal complement of N(A')

    - Any matrix can be “diagonalized” into two square matrices whose columns contain orthonormal bases for the four fundamental subspaces of A (essentially the Singular Value Decomposition)
    - To me this overarching approach of seeing comprehensive subspace information about any matrix was novel and interesting …
      it effectively organizes solving systems of equations into one conceptual picture
- A thorough treatment of projection matrices including applications to least squares (Strang revisits them over and over again)
- An intuitive treatment of determinants (that was one area where my Binghamton course nearly 25 years ago was also good)
- An intuitive motivation for eigenvectors: those exceptional vectors that are in the same direction as the action of the matrix A, that is, $Ax = \lambda x$. [The course includes some very cool supplementary graphics tools for getting a handle on eigenvalues and eigenvectors].
- Seeing why the spectral theorem is also called “the principle axis theorem” by computing the ellipsoid given by the quadratic form x’Ax (the same form used to assess positive definite matrices)
- Presenting the spectral theorem for complex matrices as well
- Seeing the matrix approach (A=LDL’) to “completing the square” that works with quadratic forms with n variables
- Categorizing matrix actions (linear transformations) by identifying types of matrices (symmetric, orthogonal, skew-symmetric, hermetian, markov, projection, rank one, etc.)
- Singular Value Decomposition (SVD) which Strang calls the “highlight” of the course: how to factor any matrix
- Right and Left inverses; pseudoinverses
- The idea of fundamental matrix factorizations as actions that “record” the essence of the algorithms of linear algebra
    - Gaussian elimation as a technique that executes the matix action (or factorization) PA = LU where P is a permutation matrix, L is the lower triangular matrix of multipliers used in elimation and U is the upper triangular result of elimination
    - A = QR for Gram-Schmidt (Q is orthonormal)
    - The spectral theorem: $A = Q\Lambda Q^{-1}$ or $A = U\Lambda U^{-1}$ for complex matrices; U is unitary
    - SVD

Were these the features of Strang’s course that stood out to you? Did I miss anything that you particularly liked about the course? Did I capture the ideas correctly?

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3 Responses to “Study Linear Algebra at MIT with Gilbert Strang”

1. **Kuldeep Singh** on 26 June 2011 at 9:41 am

   

   Fantastic review. I agree with most of your comments. However I found the videos infinitely better than the book.

   Reply

2. **Roger Tobie** on 3 June 2015 at 7:54 pm

   

   Very useful, CJ, four years after you posted it. I’m a little slow sometimes. Someone else recommended Strang’s linear algebra course to me today and I found this while cleaning up old emails. Call it a confirmation.

   Reply

3. **Infomatica Academy** on 4 November 2017 at 1:29 am

   

   Thank you very much,
   You have given an important information about Study Linear Algebra, It will be very helpful for us for further study. Please keep it up, all the best. Thank you once again.

   Reply

4.