The Extraordinary “Experiments in Topology” by Stephen Barr

Posted on 17 September 2018 by **cjf**

Stephen Barr’s “Experiments in Topology” (originally published in 1964, reprinted in 1989 by Dover) is extraordinary because it treats a sophisticated mathematical subject with accessible language that can be understood by motivated junior high students. It is extraordinary because its wonderful and copious figures are remarkably clear and elucidating. It is extraordinary because it captures the flavor, depth, and breadth of a very subtle subject with carefully written passages that boil down significant complications into understandable overviews. It is extraordinary because he gives a concrete enough treatment that the attentive reader can learn something substantial of the subject while the mysteries that are both alluded to and implicit may drive the curious reader to explore its nooks and crannies. It is extraordinary because its learning-by-doing (experimental / exploring) style is infectious and the reader may be emboldened to ask their own questions and attempt their own experiments. It is extraordinary because its effective survey of key ideas in the major branches of topology make it a useful reference. It is extraordinary because it can reward the casual reader with some basic guideposts for apprehending an advanced subject while the serious student who builds all its models and tries to understand their integrated significance can extract many deeper insights from the text.

I prefer this latter, in-depth, approach and organized, through Math Counts, a mathematics group in Philadelphia, 11 deep explorations on topics from the book each with a group of 6-10 mathematically-oriented colleagues. We thoroughly enjoyed the book and our explorations, although after 11 months we wanted to move on to other ideas.

The book uses homeomorphism as its first principle for exploring topology. Barr gives several definitions, but I found the characterization on page 5 to be the most helpful: “Any distortion is allowed provided the end result is connected in the same way as the original.” This exemplifies the kind of careful but informal style of the book.

In chapters 2-6 we are introduced to the topological surfaces that can be built with a sheet of paper, namely, the plane, cylinder, torus, Möbius band, Klein bottle, and projective plane. This might be called the piecewise linear topology of 2 dimensions and it provides an introduction to ideas from geometric topology and differential topology. Barr introduces algebraic topology with Euler’s formula in chapter 1 and Betti numbers in chapter 8. The treatment of graph theory in chapters 7-8 includes an effective overview of the four color theorem which one of our participants, Kurt

Tekolste, observed follows some of the ideas of the actual proof (which was first announced in 1976, 12 years after the publication of the book). Barr includes a little bit of knot theory in chapters 6, 8, and 9. Chapter 9 on the eversion, or inside-outing, of the torus is a wonderful further introduction to differential topology. The basic ideas of point-set topology are introduced in chapters 10-11.

In short, the book gives a competent introduction to the material and the flavor of a broad swath of the main subfields of topology though Barr does not use the academic disciplinary names preferring an informal treatment. I would like to thank George Zipperlen for pointing out the breadth of coverage in the book and for providing us a lot of useful context about the various branches of topology.

For each of our monthly meetings, I prepared a set of exercises on a bite-sized part of the book (my questions were sometimes a bit off because I had never studied topology in any depth before). Working through those questions, both in preparation before a meeting and during each group session, took more time than I expected. We came to realize that beneath the surface of Barr's elementary presentation live many subtleties which neither the book nor our mathematically experienced participants were able to easily explain to each other. We had doubts about the ideas and the language in the book. It took a lot of effort to integrate each of our partial insights into apprehension. In fact, it took so much time and effort that we usually failed to complete the prepared exercise set, so the next month's frequently overlapped significantly. But in every instance we came to understand the posed questions and the book in a deeper way. And we got to enjoy the rewards of determined exploration: realization and discovery over and over again! This in-depth process convinced us that Barr's writing is very good: once we understood what he was saying and what the often unstated caveats were, we were in awe at how effectively he spoke mathematical truth in language accessible to middle school students!

Although I agree with Bruce Trumbo's assessment that much of Barr's language will cause difficulties for the reader, I think that applies to all mathematics writing: unfamiliar language about unfamiliar subjects is always difficult to interpret especially when the precision of mathematical thinking is desired. It was the general assessment of our group that overall Barr's writing is precise and nuanced despite its informal style. It is topology itself that is almost too complicated for such an elementary treatment! Barr should be commended for his brilliant success in introducing such a subtle subject to a broad population of readers while maintaining the integrity of mathematical precision even though it may take substantial effort for readers to understand many of its passages in any depth, let alone the integrated significance of them into the broad enterprise that is modern topology. However, it should be emphasized that the book rewards less intense study by giving a feeling for the subject even if you choose not to fully untangle the nuances in the guideposts provided by the text.

One delight in the book, which George Zipperlen observed, is how the chapter on the conical Möbius strip subtly motivates the cross-cap. Indeed, the conical Möbius strip, in the limit, is the cross-cap. At our June 2018 event, George said "Be patient with Barr, what seems like a side excursion will come back." Indeed, I suspect that most of the juxtapositions and suggestions of the text are meaningful even though we did not have the wherewithal to explore very many of them: it is a rich book and a rich subject!

Not only that, but your own ideas can lead you to fascinating tangents that are not in the book. One of our participants, David Sternman, imagined cutting a torus along a Möbius strip which led Michael Reichner to post a link that led us to George Hart's great demonstration video of how to cut a bagel for a better breakfast. And Jeannie Moberly impressed us with her wonderful construction of Chris Hilder's Boy's Surface model.

We noticed very few errors in the Dover edition. One of our participants, Bob Miller, helped us realize there is a typo on pages 47-48 where it says "bringing B and B' together and joining BC to B'C' " it should say "bringing B and B' together and joining B'C to BC' ". On page 91, the phrase "is not is no worse" should simply be "is no worse". On page 138, "Laocoön statute" should be "Laocoön statue". In Fig. 25 on page 144, the dashed line "y" should be external and not internal to the inside-out torus.

Here is a list of links to the event descriptions and associated exercise sets for each of the 11 events I organized on Barr's book (several of the comments posted to these events are also interesting):

1. Topological Surfaces from a Strip of Paper (feat. minimal length Möbius strip) (Oct 2017)
    - 26 exercises in support of reading and discussing chapters 1-3
2. Variations of the Möbius Band to Explore the Nature of Homeomorphism (Nov 2017)
    - 27 exercises in support of reading and discussing chapter 3-4
3. Exploring Homeomorphism through Experiments on the Möbius Band (Dec 2017)"
    - 23 exercises in support of reading and discussing chapters 3-4
4. Topological Experiments: The Conical Möbius Band & the Klein Bottle (Jan 2018)
    - 20 exercises in support of reading and discussing chapter 4 pages 50-61, chapter 2 pages 34-39, and chapter 5 pages 62-69
5. Experiments in Topology: Dissecting The Klein Bottle (Feb 2018)
    - 20 exercises in support of reading and discussing chapter 5
6. Exploring the Topology of the Projective Plane (Mar 2018)
    - 22 exercises in support of reading and discussing chapter 2 pages 34,38 and chapter 6 pages 78-107
7. Map Coloring; Martin Gardner's Projective Plane & variations (Apr 2018)
    - 11 excercises in support of reading and discussing chapter 7 and chapter 6 pages 82-91
8. The Symmetry of the Projective Plane (and the curious property of twist) (May 2018)
    - 15 exercises in support of reading and discussing chapter 6 pages 82-107
9. Using Twist to Understand the Topology of the Projective Plane & its Symmetry (Jun 2018)
    - 11 exercises in support of reading and discussing chapter 6 pages 82-107
10. Betti Numbers and the Symmetry of the Projective Plane (Jul 2018)
    - 10 exercises in support of reading and discussing chapter 8 pages 123-128 (on Betti numbers) and chapter 6 pages 96-107 (on the projective plane)
11. Deliberations in The Trial of the Punctured Torus (Aug 2018)
    - Note: 13 exercises on chapter 9 are included in the event description instead of a separate PDF

In my research on Barr's book, I found these additional resources:

- Dover's product page for the book. Dover also provides a PDF of chapter 1 of the book.

Topological Surfaces from a Strip of Paper (feat. minimal length Möbius strip)

Hosted By
CJ F. and Sam B.

Details

What can we learn about the basic concepts of topology from paper representations of topological surfaces? Guided by chapters 1-3 of Stephen Barr's fun book Experiments in Topology, we will explore topological surfaces (https://en.wikipedia.org/wiki/Surface_(topology)) through the device of building paper models of them. Through this process we will explore several basic notions of topology at an elementary level including homeomorphisms (https://en.wikipedia.org/wiki/Homeomorphism), simply connected surfaces (https://en.wikipedia.org/wiki/Simply_connected_space), Euler's formula (https://en.wikipedia.org/wiki/Euler_characteristic), and orientability (https://en.wikipedia.org/wiki/Orientability).

To help guide the discussion and to help focus your efforts reading the book and building the paper models it describes, I have prepared a list of 26 questions and problems (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.01.pdf).

Selected questions for the discussion from the full list (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.01.pdf):

• According to Barr's book, what is the rule for homeomorphically distorting one surface into another?

• Compared to a topologically ideal surface, what properties does a sheet of paper have that make it inherently non-topological?

• What artistry can we employ in the building of models of topological surfaces from strips of paper to rectify these deficiencies, at least to some extent?

• What topological properties are impossible to represent with a physical sheet of paper as a model of a topological surface?

• How can you represent a topological plane, cylinder, torus, and Möbius strip with a strip or sheet of paper?

• Which of the following surfaces are simply connected: the sphere, a plane, a cylinder, a torus, an annulus, a Möbius strip, a mug with a handle, and a disk (the 2D surface bounded by a circle)? Why?

• What is the topological property of orientability? Which of the topological sphere, plane, cylinder, torus, and Möbius strip are orientable and which are nonorientable? Do your paper models of these topological surfaces exhibit the orientable property? How?

• Which of the following topological surfaces are homeomorphic to another surface in the list: a sphere (as a 2D surface), a plane, a cylinder, an annulus, a torus, a Möbius strip, a mug with a handle, and a disk? Why?

• In a paper model of a Möbius strip, what happens when you cut through the middle of the strip? What are the resulting surface(s)? How many sides, edges, and separate pieces are there? What logic explains this behavior?

• Can one build a paper model of a Möbius strip whose width is greater than its length? That is, can the width to length ratio (width/length) ever exceed 1?

• What is the maximum width to length ratio for a paper model of a Möbius strip?

• How can one build such a "minimum length" Möbius strip from paper?

• What is the width to length ratio in your "minimal length" paper Möbius strip model(s)?

• What is width to length ratio of the minimal length paper Möbius strip which can still be cut through the middle of the strip? Why does this property of cutting a Möbius strip through the middle fail when the width to length ratio exceeds that amount?

To prepare for the discussion, it is recommended that you read chapters 1-3 in Stephen Barr's book "Experiments in Topology" and think about the full list of 26 questions and problems prepared for the event (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.01.pdf).

More information about Stephen Barr's book "Experiments in Topology":

• "Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)

• "Experiments in Topology" at Google Books (https://books.google.com/books/about/Experiments_in_Topology.html?id=KFeGa7Ok954C)

- H. M. Cundy's review in The Mathematical Gazette Vol. 50, No. 373 (Oct., 1966), pp. 323-324.
- Bruce Trumbo's review in Mathematics Magazine Vol. 38, No. 1 (Jan., 1965), p. 50.
- Karen M. Daniels of the University of Massachusetts Lowell has 14 slides based on Barr's book to introduce topology to her Graduate Geometric Modeling students.
- Maria E. Salcedo's paper "Knotted Ribbons and Ribbon Length" includes a section on Barr's treatment of the shortest Möbius strip.

- Matthew R. Francis has three blog posts on the book: Hearing Around Corners, Simply Connect (Further Adventures in Topology), and Time to Make the Paper Donuts.
- Alexander Bogomolny's Cut-The-Knot site has a landing page with a brief review of the book and a copy of its table of contents and back cover.

I have found little biographical information about Stephen Barr. If you know anything about him or if you know of another good resource on the book or have some experience or question about it, please post a comment as I'd love to learn more.

Whoever Stephen Barr is or was, the book "Experiments in Topology" is an able tour guide to lead both casual and serious readers to apprehend some key ideas in topology, one of the most storied and important branches of modern mathematics.

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Discussion Questions and Problems for "Topological Surfaces from a

Strip of Paper"

(Revision: October 26, 20171)

On 28 October 2017 Math Counts will discuss Topological Surfaces from a Strip of Paper2. These problems are based on Chapters 1-3 of Stephen Barr's fun 238 page book "Experiments in Topology". Your attempts to address these questions and problems will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page2.

1. According to Barr's book, what is the rule for homeomorphically distorting one surface into another?

2. Compared to a topologically ideal surface, what properties does a sheet of paper have that make it inherently non-topological?

3. What artistry can we employ in the building of models of topological surfaces from strips of paper to rectify these deficiencies, at least to some extent?

4. What topological properties are impossible to represent with a physical sheet of paper as a model of a topological surface?

5. Do you agree with Stephen Barr that it is impossible to represent a topological sphere (connsidered as a 2D surface) with a sheet of paper? Why? Why not?

6. How can you represent a topological plane, cylinder, torus, and M¨obius strip with a strip or sheet of paper?

7. How many joints (taping or glueing or otherwise joining of edges together) are needed in a paper representation of a plane, cylinder, torus, and Mobius strip?

8. If a sheet of paper is said to have 2 sides (faces or surfaces or 2D extents: the front and back, for instance) and 4 edges (the top, bottom, left and right), how many sides and edges are in a paper model of a plane, cylinder, torus, and Mobius strip?

9. What are the numbers of vertices, edges, and faces (sides) in a paper model of a plane, cylinder, torus, and Mobius strip? Compute F-E+V for each of your paper models where F represents the number of faces or sides, E is the number of edges or joints where two faces

meet, and V represents the number of vertices or crossings where two or more edges meet.

10. Which of the following surfaces are simply connected: the sphere, a plane, a cylinder, a torus, an annulus, a Mobius strip, a mug with a handle, and a disk (the 2D surface bounded by a circle)? Why?

11. What is the topological property of orientability? Which of the topological sphere, plane, cylinder, torus, and Mobius strip are orientable and which are nonorientable? Do your paper models of these topological surfaces exhibit the orientable property? How?

1Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found.

Compiled by CJ Fearnley. http://blog.CJFearnley.com.

2http://www.meetup.com/MathCounts/events/243186341

12. Which of the following topological surfaces are homeomorphic to another surface in the list: a sphere (as a 2D surface), a plane, a cylinder, an annulus, a torus, a Mobius strip, a mug with a handle, and a disk? Why?

13. In a paper model of a Mobius strip, what happens when you cut through the middle of the strip? What are the resulting surface(s)? How many sides, edges, and separate pieces are there? What logic explains this behavior?

14. Can one build a paper model of a Mobius strip whose width is greater than its length (let us define the width to be the two opposite ends of the rectangular strip of paper that are joined along their full extent when making the Mobius band)? That is, can the width to length ratio (width/length) ever exceed 1?

15. In the Mobius strip construction using a folded 60◦ degree angle to form a hexagon shown in

Figure 4 of chapter 3 on p. 42, what is the ratio of width to length?

16. In the Mobius strip construction in Figure 7 on p. 43, what is the width to length ratio?

17. In the Mobius strip construction in Figure 9 on p. 44, what is the width to length ratio?

18. In the Mobius strip construction in Figure 13 on p. 46, what is the width to

length ratio?

19. In the Mobius strip construction in Figure 14 on p. 47, what is the width to length ratio?

20. How can one build a paper model of a Mobius strip from a square strip of paper?

21. What is the maximum width to length ratio for a paper model of a Mobius strip?

22. How can one build such a “minimum length” Mobius strip from paper?

23. What is the width to length ratio in your “minimal length” paper Mobius strip model(s)?

24. What is the width to length ratio of the minimal length paper M¨obius strip which can still be cut through the middle of the strip? Why does this property of cutting a Mobius strip through the middle fail when the width to length ratio exceeds that amount?

25. In considering the sequence of experiments discussed in chapter 3 where Mobius strips of decreasing length to width ratios (or increasing width to length ratios) are considered, what did you learn about the nature of topology, geometry, and paper representations of topological surfaces?

26. In the text on p. 49 and the associated Figures 15 and 16, a little puzzle about cutting a Mobius strip into two equal area pieces is described. What is the solution to the puzzle? Why does the cut line shown in Figure 16 fail to work as intended? What does that cut actually do? Why must the cut start on the edge? What does it mean for the cut to start on the edge? What logic explains the puzzle? What did you learn from this puzzle?

Variations of the Möbius Band to Explore the Nature of Homeomorphism

Hosted By
CJ F. and Sam B.

Details

Stephen Barr defines a homeomorphism (https://en.wikipedia.org/wiki/Homeomorphism) with a "no-cutting-or-joining rule": "distortions are only allowed if one does not disconnect what was connected (like making a cut or a hole), nor connect what was not (like joining the ends of the previously unjoined string, or filling in the hole)". It is an intuitively conceptual definition instead of a mathematically rigorous one. Its subtleties require a deeper exploration.

This session will explore the nature of homeomorphism by considering several variations on a Möbius strip made from a strip of paper.

In particular, we will consider the minimal length Möbius band. How much distortion is permitted in a homeomorphism of a Möbius strip? What does connectedness mean in a highly folded model of a Möbius band?

In addition, we will consider the conical Möbius band. How much of the edges of a strip of paper must be joined to properly produce a Möbius band?

Our guide for exploring these mind-bending ways to imagine homeomorphism is chapters 3-4 of Stephen Barr's fun book Experiments in Topology. A review of chapters 1 and 2 is recommended for newcomers and for all participants to deepen our understanding of the basic concepts.

It is recommended that participants build as many of the paper models discussed in the book as they have time for. Building the models and thinking about them topologically will help with our task of exploring the subject during our discussion.

To further guide your thinking about the variations on a Möbius strip that we will discuss, I organized this list of 27 questions (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.02.pdf) to guide your preparations in thinking about the book.

Here are some selected questions for the discussion from the full list (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.02.pdf):

• According to chapter 1 of Barr's book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr's definition compare to Wikipedia's definition of homeomorphism?

• According to chapter 1 of Barr's book, what is the definition for a simply connected surface? How does Barr's definition compare to Wikipedia's definition of a simply connected space?

• What is the property of orientability? Which of the sphere, disk (the 2D surface bounded by a circle), cylinder, torus, and Möbius band are orientable and which are nonorientable? How can you explain orientability with paper models of topological surfaces?

• In a paper model of a Möbius strip, what happens when you cut through the middle of the strip? What are the resulting surface(s)? How many sides, edges, and separate pieces are there? What logic explains this behavior?

• Can one build a paper model of a Möbius strip whose width is greater than its length (let us define the width to be the length of each of the two opposite edges of the rectangular strip of paper that are glued together when making the Möbius band)? That is, can the width to length ratio (width/length) ever exceed 1? Note: the length of a Möbius band might be said to be the length of its one and only edge.

• How can one build a paper model of a Möbius band from a square strip of paper?

• What is the maximum width to length ratio for a paper model of a Möbius strip?

• How can one build such a "minimum length" Möbius band from paper?

• What is the width to length ratio of the minimal length paper Möbius strip which can still be cut along the centerline of the strip? Why does the property of cutting a Möbius strip through its centerline fail when the width to length ratio exceeds that amount?

• In considering the variations on a Möbius band in chapter 3, how should we think about the connectedness of the joined edge of a Möbius band? How is the homeomorphic property preserved in each of these variations?

• In considering the sequence of experiments in chapter 3 where Möbius strips of decreasing length to width ratios (or increasing width to length ratios) are considered, what did you learn about the nature of homeomorphism and paper representations of topological surfaces?

• In considering the variations on a Möbius band in chapter 4, how should we think about the distortion and connectedness in the joined edge of the strip of paper used to make a Möbius band? How is the homeomorphic property preserved in each of these variations?

• In considering the sequence of experiments discussed in chapter 4 where Möbius strips with various extents of connectedness are considered, what did you learn about the nature of homeomorphism and paper representations of topological surfaces?

• How do the properties of simply connected surfaces and orientability play a role in understanding the variations on a Möbius band considered in chapters 3 and 4?

• Given the considerations in chapters 3 and 4, what subtleties, limitations, and caveats must we heed about the nature of the distortions allowed and the requirements of connectedness and continuity in Stephen Barr's definition of a homeomorphism?

This meetup is part of a series exploring the content of Stephen Barr's fun book "Experiments in Topology".

"Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)

"Experiments in Topology" at Google Books (https://books.google.com/books?id=9TMx6ABV-98C)

Discussion Questions and Problems for “Variations of the M¨obius

Band to Explore the Nature of Homeomorphism”

(Revision: November 12, 20171)

On 25 November 2017 Math Counts will discuss “Variations of the M¨obius Band to Explore the Nature of Homeomorphism”2. These problems are based on Chapters 3-4 of Stephen Barr’s fun book “Experiments in Topology”. Your attempts to address these questions and problems will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page2.

1. According to chapter 1 of Barr's book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr's definition compare to Wikipedia's definition of homeomorphism?

2. According to chapter 1 of Barr's book, what is the definition for a simply connected surface? How does Barr's definition compare to Wikipedia's definition of a simply connected space?

3. What is the property of orientability? Which of the sphere, disk (the 2D surface bounded by a circle), cylinder, torus, and Mobius band are orientable and which are nonorientable? How can you explain orientability with paper models of topological surfaces?

4. In a paper model of a Mobius strip, what happens when you cut through the middle of the strip? What are the resulting surface(s)? How many sides, edges, and separate pieces are there? What logic explains this behavior?

5. Can one build a paper model of a Mobius strip whose width is greater than its length (let us define the width to be the length of each of the two opposite edges of the rectangular strip of paper that are glued together when making the Mobius band)? That is, can the width to length ratio (width/length) ever exceed 1? Note: the length of a M¨obius band might be said to be the length of its one and only edge.

6. In the Mobius strip construction using a folded 60◦ degree angle to form a hexagon shown in Figures 4 and 5 of chapter 3 on p. 42, what is the ratio of width to length?

7. In the Mobius strip construction in Figure 7 on p. 43, what is the width to length ratio?

8. In the Mobius strip construction in Figure 9 on p. 44, what is the width to length ratio?

9. In the Mobius strip construction in Figure 13 on p. 46, what is the width to length ratio?

10. In the Mobius strip construction in Figure 14 on p. 47, what is the width to length ratio?

11. How can one build a paper model of a Mobius band from a square strip of paper?

12. What is the maximum width to length ratio for a paper model of a Mobius strip?

13. How can one build such a "minimum length" Mobius band from paper?

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Compiled by CJ Fearnley. http://blog.CJFearnley.com.

2http://www.meetup.com/MathCounts/events/243186341

14. What is the width to length ratio in your "minimal length" paper Mobius strip model(s)?

15. What is the width to length ratio of the minimal length paper Mobius strip which can still be cut along the centerline of the strip (see question 4)? Why does the property of cutting a

Mobius strip through its centerline fail when the width to length ratio exceeds that amount?

16. In considering the variations on a Mobius band in chapter 3, how should we think about the connectedness of the joined edge of a Mobius band? How is the homeomorphic property preserved in each of these variations?

17. In considering the sequence of experiments in chapter 3 where Mobius strips of decreasing length to width ratios (or increasing width to length ratios) are considered, what did you learn about the nature of homeomorphism and paper representations of topological surfaces?

18. In the text on p. 49 and the associated Figures 15 and 16, a puzzle about cutting a Mobius strip into two equal area pieces is described. What is the solution to the puzzle? What does it mean for the cut to start on the edge? What logic explains the puzzle? What did you learn from this puzzle?

19. Why are the paper models of the Klein Bottle and Projective Plane in Figures 2 and 3 on p. 51 considered troublesome? Is the difficulty related to the subtleties of connectedness in the definition of a homeomorphism? How would you explain the problem?

20. Can a Mobius strip be constructed from an annulus with a radial cut through its bounding circles as in Figure 4 on p. 52?

21. Can a Mobius strip be constructed from a disk with a radial cut to its center?

22. Can a Mobius band be constructed from a semicircle as in Figure 11 on p. 55?

23. Can a Mobius strip be constructed from a 30° sector of a circle?

24. In considering the variations on a Mobius band in chapter 4, how should we think about the distortion and connectedness in the joined edge of the strip of paper used to make a Mobius band? How is the homeomorphic property preserved in each of these variations?

25. In considering the sequence of experiments discussed in chapter 4 where Mobius strips with various extents of connectedness are considered, what did you learn about the nature of homeomorphism and paper representations of topological surfaces?

26. How do the properties of simply connected surfaces and orientability play a role in understanding the variations on a Mobius band considered in chapters 3 and 4?

27. Given the considerations in chapters 3 and 4, what subtleties, limitations, and caveats must we heed about the nature of the distortions allowed and the requirements for connectedness and continuity in Stephen Barr's definition of a homeomorphism?

Exploring Homeomorphism through Experiments on the Möbius Band

Hosted By
CJ F. and Sam B.

Details

We will explore the nature of homeomorphism by considering several variations on a Möbius strip made from paper.

Stephen Barr, in his fun book "Experiments in Topology", defines a homeomorphism (https://en.wikipedia.org/wiki/Homeomorphism) with a "no-cutting-or-joining rule": "distortions are only allowed if one does not disconnect what was connected (like making a cut or a hole), nor connect what was not (like joining the ends of the previously unjoined string, or filling in the hole)". It is an intuitively conceptual definition instead of a mathematically rigorous one. Its subtleties require a deeper exploration.

In particular, we will consider the minimal length Möbius band. How much distortion is permitted in a homeomorphism of a Möbius strip? What does connectedness mean in a highly folded model of a Möbius band?

In addition, we will consider the conical Möbius band. How much of the edges of a strip of paper must be joined to properly produce a Möbius band?

Our guide for exploring these mind-bending ways to imagine homeomorphism is chapters 3-4 of Stephen Barr's fun book Experiments in Topology. A review of chapters 1 and 2 is recommended for newcomers and for all participants to deepen our understanding of the basic concepts.

It is recommended that participants build as many of the paper models discussed in the book as they have time for. Building the models and thinking about them topologically will help with our task of exploring the subject during our discussion.

To further guide your thinking about the experiments on a Möbius strip that we will discuss, I have organized this list of 23 questions (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.03.pdf) to guide your preparations in thinking about the book.

Here are some selected questions for the discussion from the full list (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.03.pdf):

• According to chapter 1 of Barr's book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr's definition compare to Wikipedia's definition of homeomorphism?

• What is the property of orientability? Which of the sphere, disk (the 2D surface bounded by a circle), cylinder, torus, and Möbius band are orientable and which are nonorientable?

• In a paper model of a Möbius strip, what happens when you cut through the middle of the strip? How would you describe the resulting surface? What familiar surface is the result homeomorphic to?

• Can one build a paper model of a Möbius strip whose width is greater than its length? Let us define the width of a Möbius band to be the length of either of the two opposite edges that are glued together when making the Möbius band. Let us define its length to be the length of its one and only edge. So, can the width to length ratio (width/length) ever exceed 1 (or even ½)?

• What is the maximum width to length ratio possible for a Möbius strip?

• Build a Möbius strip with the largest width to length ratio you have the patience and wherewithal to make. What is the width to length ratio in your model?

• What is the width to length ratio of the minimal length paper Möbius strip which can still be cut along the centerline of the strip? Why does the property of cutting a Möbius strip through its centerline fail when the width to length ratio exceeds 1/2?

• In considering the variations on a Möbius band in chapter 3, how should we think about the connectedness of the joined edge of a Möbius band? How is the homeomorphic property preserved in each of these variations?

• In considering the sequence of experiments in chapter 3 where Möbius strips of decreasing length to width ratios (or increasing width to length ratios) are considered, what did you learn about the nature of homeomorphism and paper representations of topological surfaces?

• In considering the variations on a Möbius band in chapter 4, how should we think about the distortion and connectedness in the joined edge of the strip of paper used to make a Möbius band? How is the homeomorphic property preserved in each of these variations?

• In considering the sequence of experiments discussed in chapter 4 where Möbius strips with various extents of connectedness are considered, what did you learn about the nature of homeomorphism and paper representations of topological surfaces?

• Given the considerations in chapters 3 and 4, what subtleties, limitations, and caveats must we heed about the nature of the distortions allowed and the requirements of connectedness and continuity in Stephen Barr's definition of a homeomorphism?

• Inspired by the book or these questions or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

This meetup is part of a series exploring the content of Stephen Barr's fun book "Experiments in Topology".

"Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)

"Experiments in Topology" at Google Books (https://books.google.com/books?id=9TMx6ABV-98C)

We strive to make each event accessible to newcomers. Key concepts will be reviewed and an effort to explain any technical terms will be made. If anything is unfamiliar to you, please ask and we will try to clarify.

Discussion Questions and Problems for “Exploring Homeomorphism

through Experiments on the M¨obius Band”

(Revision: December 19, 20171)

On 23 December 2017 Math Counts will discuss “Exploring Homeomorphism through Experiments on the M¨obius Band”2. The following questions and problems are based on Chapters 3-4 of Stephen

Barr’s fun book “Experiments in Topology”. Your attempts to address them will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page2.

1. According to chapter 1 of Barr’s book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr’s definition compare to Wikipedia’s definition of homeomorphism?

2. What is the property of orientability? Which of the sphere, disk (the 2D surface bounded by a circle), cylinder, torus, and M¨obius band are orientable and which are nonorientable?

3. In a paper model of a M¨obius strip, what happens when you cut through the middle of the strip? How would you describe the resulting surface? What familiar surface is the result homeomorphic to?

4. Can one build a paper model of a Mobius strip whose width is greater than its length? Let us define the width of a Mobius band to be the length of either of its two opposite edges that are glued together. Let us define its length to be the length of its one and only edge. So, can the width to length ratio (width/length) ever exceed 1 (or even $\frac{1}{2}$)?

5. In the Mobius strip construction with three equilateral triangles in Figure 7 on p. 43, what is the width to length ratio?

6. In the Mobius strip construction in Figure 14 on p. 47, what is the width to length ratio? During the Nov 25th meetup, we were unable to justify the width to length result given by Barr on p. 48. Is he mistaken? What is going on with this Mobius strip?

7. What is the maximum width to length ratio possible for a Mobius strip?

8. Build a Mobius strip with the largest width to length ratio you have the patience and wherewithal to make. What is the width to length ratio in your model?

9. On p. 48, Barr suggests that the Mobius band folded from a square strip of paper has the minimal width to length ratio which can still be cut along the centerline of the strip and

unfolded to a new surface (see question 3)? Why does the property of cutting a Mobius strip through its centerline fail when the width to length ratio exceeds 1

2 ?

10. In the text on p. 49 and the associated Figures 15 and 16, a puzzle about cutting a Mobius strip into two equal area pieces is described. What is the solution to the puzzle? What does it mean for the cut to start on the edge? What logic explains the puzzle? What did you learn from this puzzle?

[1]Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found.

Compiled by CJ Fearnley. http://blog.CJFearnley.com.

[2]http://www.meetup.com/MathCounts/events/243186341

11. In considering the variations on a Mobius band in chapter 3, how should we think about the connectedness of the joined (glued) edge of a Mobius band? How is the homeomorphic property preserved in each of these variations? Are you sure? Why?

12. In considering the sequence of experiments in chapter 3 where Mobius strips of decreasing length to width ratios (or increasing width to length ratios) are considered, what did you learn about the nature of homeomorphism and paper representations of topological surfaces?

13. Why are the paper models of the Klein Bottle and Projective Plane in Figures 2 and 3 on p. 51 considered troublesome? Is the difficulty related to the subtleties of connectedness in the definition of a homeomorphism? How would you explain the problem?

14. What does Barr's text mean on p. 52 where it says "some meaningful restrictions must be placed on the Mobius strip, too, as to how much of the edges ought to be joined" and in wondering "if the amount of edge involved can be increased "? What does it mean to increase or decrease the amount of edge involved? What are the restrictions alluded to?

15. How can a Mobius strip be constructed from an annulus with a radial cut through its bounding circles as in Figure 4 on p. 52? In what way does this experiment test the question about the amount of edge involved in forming a Mobius band?

16. How can a Mobius strip be constructed from a disk with a radial cut to its center? What is the width to length ratio in the resulting Mobius band?

17. How can a Mobius band be constructed from a semicircle as in Figure 11 on p. 55? What is the width to length ratio in the resulting Mobius strip?

18. How can a Mobius strip be constructed from a 30° sector of a circle? What is the width to

length ratio in the resulting Mobius band?

19. On p. 61, Barr concludes chapter 4 by saying “The moral of all this is that when we allow only one kind of distortion (bending), unexpected relationships persist. Suspicion arises that with any distortion allowed, what persists must be invariant indeed, and perhaps overlooked before.” How do you interpret this conclusion? What invariants did you infer from the experiments into topology that you undertook in reading chapters 3 and 4?

20. In considering the variations on a Mobius band in chapter 4 and any additional thought or model-building experiments you might have undertaken, how should we think about the distortion and connectedness in the joined edge of the strip of paper used to make a Mobius band? How is the homeomorphic property preserved in each of these variations?

21. In considering the sequence of experiments discussed in chapter 4 where Mobius strips with various extents of connectedness are considered, what did you learn about the nature of homeomorphism, topological invariants, and paper representations of topological surfaces?

22. Given the considerations in chapters 3 and 4, what subtleties, limitations, and caveats must we heed about the nature of the distortions allowed and the requirements for connectedness and continuity in Stephen Barr’s definition of a homeomorphism?

23. Inspired by the book or these questions or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

Topological Experiments: The Conical Möbius Band & the Klein Bottle

Hosted By
CJ F. and Sam B.

Details

Stephen Barr's fun book "Experiments in Topology" gives a feeling for homeomorphism (a fundamental concept in topology) with paper models of Möbius Bands and Klein Bottles (and more). Our task in this meetup is to explore The Conical Möbius Strip (Chapter 4 pp. 50-61) and The Klein Bottle (Chapter 2 pp. 34-39 and the first half of Chapter 5 pp. 62-69) in the book. The introductory section of Chapter 1 (pp. 1-9) and the section in Chapter 2 on orientability (pp. 25-28) provide important background information for this session.

On p. 4, Barr defines a homeomorphism (https://en.wikipedia.org/wiki/Homeomorphism) with a rule: "distortions are only allowed if one does not disconnect what was connected (like making a cut or a hole), nor connect what was not (like joining the ends of the previously unjoined string, or filling in the hole)". On p. 5 he simplifies this "no-cutting-or-joining rule": "Any distortion is allowed provided the end result is connected in the same way as the original". It is an intuitively conceptual definition instead of a mathematically rigorous one. Experimentation is needed to develop an intuition for its meaning.

The hope and intention for this session is to build our intutions about topology and homeomorphism through exploring paper models of the Möbius band and the Klein bottle.

It is recommended that participants build as many of the paper models discussed in the book as they have time for. Building the models and thinking about them ahead of time will help with our task of exploring the subject during our discussion.

To guide your exploration of the text and to guide our exploration during the meetup, I have organized this list of 20 questions:
http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.04.pdf

Here are some selected questions for the discussion from the full list (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.04.pdf):

● Inspired by the book, the questions that follow or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

● In considering the variations on a Möbius band in chapter 4 and any additional thought or model-building experiments you might have undertaken, how should we think about the distortion and connectedness in the joined edge of the strip of paper used to make a Möbius band? How is the homeomorphic property preserved in each of these variations?

● According to chapter 1 of Barr's book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr's definition compare to Wikipedia's definition of homeomorphism?

● What is the property of orientability? Which of the sphere, disk (the 2D surface bounded by a circle), cylinder, torus, and Möbius band are orientable and which are nonorientable?

● After reading chapter 4, can you explain what Barr's text means on p. 52 where it says "some meaningful restrictions must be placed on the Möbius strip, too, as to how much of the edges ought to be joined" and in wondering "if the amount of edge involved can be increased"? What does it mean to increase or decrease the amount of edge involved? What are the restrictions alluded to?

● On p. 61, Barr concludes chapter 4 by saying "The moral of all this is that when we allow only one kind of distortion (bending), unexpected relationships persist. Suspicion arises that with any distortion allowed, what persists must be invariant indeed, and perhaps overlooked before." How do you interpret this conclusion? What invariants did you infer from the experiments into topology that you undertook in reading chapter 4?

● In considering the variations on a Möbius band in chapter 4 and any additional thought or model-building experiments you might have undertaken, how should we think about the distortion and connectedness in the joined edge of the strip of paper used to make a Möbius band? How is the homeomorphic property preserved in each of these variations?

● In considering the sequence of experiments discussed in chapter 4 where Möbius strips with various extents of connectedness are considered, what did you learn about the nature of homeomorphism, topological invariants, and paper representations of topological surfaces?

● Given the considerations in chapter 4, what subtleties, limitations, and caveats must we heed about the nature of the distortions allowed and the requirements for connectedness and continuity in Stephen Barr's definition of a homeomorphism?

● What happens when the Klein bottle is cut in two?

● In considering the conical Möbius band and the Klein bottle and two of its dissections in the first half of chapter 5, what observations, realizations, or insights have you acquired about the nature of topology, homeomorphism, orientability, and the connectedness of topological surfaces?

This meetup is part of a series exploring the content of Stephen Barr's fun book "Experiments in Topology".

"Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)

"Experiments in Topology" at Google Books (https://books.google.com/books?id=9TMx6ABV-98C)

Each event will be made accessible to newcomers. Key concepts will be reviewed and an effort to explain technical terms will be made. If anything is unfamiliar to you, please ask and we will try to clarify.

Discussion Questions and Problems for “Topological Experiments: The Conical Mobius Band & the Klein Bottle”

(Revision: January 19, 20181)

On 27 January 2018 Math Counts will discuss “Topological Experiments: The Conical Mobius Band & the Klein Bottle”2. The following questions and problems are based on Stephen Barr’s fun book “Experiments in Topology”. Your attempts to address them will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page2.

1. Inspired by the book, the questions that follow or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

2. According to chapter 1 of Barr’s book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr’s definition compare to Wikipedia’s definition of homeomorphism?

3. What is the property of orientability? Which of the sphere, disk (the 2D surface bounded by a circle), cylinder, torus, and Mobius band are orientable and which are nonorientable?

4. Why are the paper models of the Klein Bottle and Projective Plane in Figures 2 and 3 on p. 51 considered troublesome? Is the difficulty related to the subtleties of connectedness in the definition of a homeomorphism? How would you explain the problem?

5. How can a Mobius strip be constructed from an annulus with a radial cut as in Figure 4 on p. 52? In what way does this experiment test the question about the amount of edge involved in forming a Mobius band?

6. How can a Mobius strip be constructed from a disk with a radial cut to its center? What are the lengths of the joined and unjoined edges of the resulting Mobius band? Let us define the width of a Mobius band to be the length of either of its two opposite edges that are glued together (after adding the half-twist). Let us define its length to be the length of its one and only edge. What is the width to length ratio in this Mobius strip?

7. How can a Mobius band be constructed from a semicircle as in Figure 11 on p. 55? What are the lengths of the joined and unjoined edges of the resulting Mobius strip? What is the width to length ratio in this Mobius strip?

8. How can a Mobius strip be constructed from a 30° sector of a circle? What are the lengths of the joined and unjoined edges of the resulting Mobius strip? What is the width to length ratio in this Mobius band?

9. After reading chapter 4, can you explain what Barr's text means on p. 52 where it says "some meaningful restrictions must be placed on the Mobius strip, too, as to how much of the edges ought to be joined" and in wondering "if the amount of edge involved can be increased "? What does it mean to increase or decrease the amount of edge involved? What are the restrictions alluded to?

1Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found.

Compiled by CJ Fearnley. http://blog.CJFearnley.com.

2https://www.meetup.com/MathCounts/events/246163335/

10. On p. 61, Barr concludes chapter 4 by saying "The moral of all this is that when we allow only one kind of distortion (bending), unexpected relationships persist. Suspicion arises that with any distortion allowed, what persists must be invariant indeed, and perhaps overlooked before." How do you interpret this conclusion? What invariants did you infer from reading chapter 4 and any experiments you undertook?

11. In considering the variations on a Mobius band in chapter 4 and any additional thought

or model-building experiments you might have undertaken, how should we think about the distortion and connectedness in the joined edge of the strip of paper used to make a Mobius band? How is the homeomorphic property preserved in each of these variations?

12. In considering the sequence of experiments discussed in chapter 4 where Mobius strips with various extents of connectedness are considered, what did you learn about the nature of homeomorphism, topological invariants, and paper representations of topological surfaces?

13. Given the considerations in chapter 4, what subtleties, limitations, and caveats must we heed about the nature of the distortions allowed and the requirements for connectedness and continuity in Stephen Barr's definition of a homeomorphism?

14. In the description of the Klein bottle on pp. 34–35, the order of joining the opposite edges is discussed. Why is it that it seems impossible to make the half-twist Mobius join first? What is it about the order of operations that makes doing the untwisted join first easier to imagine?

15. In Fig. 22 and 23 on p. 37 and the surrounding text, a way to build a symmetrical Klein bottle is described. How can you build it with paper? How should we interpret this model given the statements in the text on p. 38 that "the surface passes through itself" and "That is to say, in intersecting, neither plane interrupts the continuity of the other"?

16. How can one see and understand the nonorientability of the Klein bottle? How can you appreciate its nonorientability in a paper model? Can we understand the nonorientability from the connectedness diagram in figure 18 on p. 34? Does the text on pp. 62–63 clarify the matter?

17. On p. 63, the book asks "What happens when a Klein bottle is cut in two?" What do you think? How many cases need to be considered?

18. How can you interpret the model in Fig. 5 on p. 65 as a Klein bottle? If we regard the joint CC′ as a cut, how can you interpret the resulting model? How would you describe that cut and its results?

19. How can you interpret the model in Fig. 10 on p. 68 as a Klein bottle? If we regard the joint between the edges AB′ and A′B as a cut, how can you interpret the resulting model? How would you describe that cut and its results?

20. In considering the conical Mobius band and the Klein bottle and two of its dissections at the beginning of chapter 5, what observations, realizations, understandings, and insights have you had about the nature of topology, homeomorphism, orientability, and the connectedness of topological surfaces?

Experiments in Topology: Dissecting The Klein Bottle

Details

Stephen Barr's fun book "Experiments in Topology" helps us begin to think topologically by building and considering paper models. In this session, we will explore several dissections of the Klein bottle. In Barr's book the Klein bottle is introduced in Chapter 2 pp. 31-39. In Chapter 5 pp. 62-77, the text develops a (possibly incomplete) classification of the dissections of the Klein bottle.

We will try to understand each cut and its effects. We will try to determine the significance of the six dissections of the Klein bottle explored in the text. We will try to understand the role of cutting or dissection in topology and the topology of surfaces. If time permits, we will begin exploring the projective plane in Chapter 6 pp. 78-107 (we will not get past p. 84 and the introduction of Martin Gardner's model for the projective plane).

The introductory section of Chapter 1 pp. 1-9 (especially the account on homeomorphism) and the beginning of Chapter 2 pp. 20-34 (especially the section on Orientability) provide important background information for this session. Since the Möbius strip is integral to an understanding of the Klein bottle, skimming Chapters 3 and 4 (especially the connectivity diagram of the Möbius strip in Fig. 15 on p. 49) might be helpful.

The hope and intention for this session is to build our intutions about topology and thinking topologically about surfaces through exploring paper models of the Klein bottle and its dissections.

It is recommended that participants build as many of the paper models discussed in the book as they have time for. Building the models and thinking about them ahead of time will help with our task of exploring the subject during our discussion.

To guide your exploration of the text and to guide our exploration during the meetup, I have organized this list of 20 questions:
http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.05.pdf .

Here are some selected questions for the discussion from the full list (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.05.pdf):

● Inspired by the book, the questions that follow or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

● According to chapter 1 of Barr's book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr's definition compare to Wikipedia's definition of homeomorphism?

● How would you describe a Klein bottle? Where is its hole? Where is its inside and its outside? What is the nature of its self-intersection? Is the self-intersection integral to the idea of a Klein bottle? Why or why not? Does the Klein bottle have a boundary (or an edge)? How would you explain the idea of a Klein bottle to a child?

● How can one see and understand the nonorientability of the Klein bottle? How can you appreciate its nonorientability in a paper model? How would you convince a child that it is nonorientable?

● What happens when the Klein bottle is cut in two? What do you think? How many cases need to be considered?

● In considering the dissections and related experiments with the Klein bottle in Chapter 5, what is the topological significance of the chapter? What did you learn about thinking topologically? What did Barr intend for us to learn?

● Given Stephen Barr's book so far, all of the experimentation and thinking you have done related to the book and our event(s), what observations, realizations, understandings, and insights have you had about the nature of topology, homeomorphism, orientability, connectedness, continuity, paper representations of topological surfaces, the nature of topological surfaces, and topological invariants?

This meetup will be part of a series exploring the content of Stephen Barr's fun book "Experiments in Topology".

"Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)"Experiments in Topology" at Google Books (https://books.google.com/books?id=9TMx6ABV-98C)

Each event will be made as accessible to newcomers as possible. Key concepts will be reviewed and an effort to explain technical terms will be made. If anything is unfamiliar to you, please ask and we will try to clarify.

Discussion Questions and Problems for “Experiments in Topology:
Dissecting The Klein Bottle”
(Revision: February 19, 20181)

1. Inspired by the book, the questions that follow, or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

2. According to chapter 1 of Barr’s book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr’s definition compare to Wikipedia’s definition of homeomorphism?

3. In the description of the Klein bottle on pp. 34–35, the order of joining the opposite edges is discussed. Why is it that it seems impossible to make the half-twist M¨obius join first? What is it about the order of operations that makes doing the untwisted join first easier to imagine?

4. How would you describe a Klein bottle? Where is its hole? Where is its inside and its outside? What is the nature of its self-intersection? Is the self-intersection integral to the idea of a

Klein bottle? Why or why not? Does the Klein bottle have a boundary (or an edge)? How would you explain the idea of a Klein bottle to a child?

5. How can one see and understand the nonorientability of the Klein bottle? How can you appreciate its nonorientability in a paper model? How can we understand the nonorientability from the connectedness diagram in figure 18 on p. 34? Does the text on pp. 62–63 clarify the matter? How would you convince a child that it is nonorientable?

6. In Fig. 22 and 23 on p. 37 and the surrounding text, a way to build a symmetrical Klein bottle is described. How can you build it with paper? How should we interpret this model given the statements in the text on p. 38 that “the surface passes through itself” and “That is to say, in intersecting, neither plane interrupts the continuity of the other”?

7. On p. 63, the book asks “What happens when a Klein bottle is cut in two?” What do you think? How many cases need to be considered?

8. How can you interpret the model in Fig. 5 on p. 65 (which is dissection #1 on p. 75) as a Klein bottle? What is joined or glued together and what is cut apart or separated or left open? If we regard the line CC′ and the join between AB′ with BA′ as cuts, how can you interpret the result? How would you describe the dissection and its results? What information about the situation can be gleaned from the top view of the main joints? And from the connectivity diagram?

1Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found.
Compiled by CJ Fearnley. http://blog.CJFearnley.com.
2https://www.meetup.com/MathCounts/events/247238814/

9. How can you interpret the model in Fig. 10 on p. 68 (dissection #2 on p. 76) as a Klein bottle? What is joined or glued together and what is cut apart or separated or left open? If we regard the joint between the edges AB′ and A′B as a cut, how can you interpret the result? How would you describe the dissection and its results? What information about the situation can be gleaned from the top view of the main joints? And from the connectivity diagram?

10. How can you interpret the model referred to in dissection #3 on p. 76 as a Klein bottle? What is joined or glued together and what is cut apart or separated or left open? In making the indicated cuts, how can you interpret the result? How would you describe the dissection and its results? What information about the situation can be gleaned from the top view of the main joints? And from the connectivity diagram?

11. Apply all of the questions in item 10 to dissection #4 on p. 76.

12. Apply all of the questions in item 10 to dissection #5 on p. 77.

13. Apply all of the questions in item 10 to dissection #6 on p. 77.

14. In considering the six dissections of the Klein bottle summarized in the list on pp. 75-77, what is the significance of the top views of the joint? And of the connectivity diagrams? What is the result of each dissection? How can we understand how each dissection produces its result? What do the dissections reveal about the topology of the Klein bottle?

15. Which of the six dissections is called the Slipped-disk Klein bottle (see p. 75)?

16. Draw lines representing cuts for each of the six dissections on the steam cabinet model (or any other 3D model) of a Klein bottle (the steam cabinet model is described on p. 72 and in the appendix on pp. 202-203)?

17. In considering the dissections and related experiments with the Klein bottle in Chapter 5, what is the topological significance of the chapter? What did you learn about thinking topologically? What did Barr intend for us to learn?

18. Given Stephen Barr's book so far, all of the experimentation and thinking you have done related to the book and our event(s), what observations, realizations, understandings, and insights have you had about the nature of topology, homeomorphism, orientability, connectedness, continuity, paper representations of topological surfaces, the nature of topological surfaces, and topological invariants?

19. Explain a cross-cap using the conceptual model described on pp. 79-82. Could you build such a model?

20. How can you interpret Martin Gardner's model in Figures 8-11 and described on pp. 83-84 as a projective plane? What is a projective plane? How does your imagination need to interpret the model to "see" the inherent self-intersection that is required in a projective plane?

Exploring the Topology of the Projective Plane

[CJ F. and Sam B.]

Details

Stephen Barr's fun book "Experiments in Topology" helps us begin to think topologically by building and considering paper models. In this session, we will explore the projective plane. In Barr's book the projective plane is introduced in Chapter 2 on pages 34 and 38 and more extensively in Chapter 6 on pages 78-107.

The introductory section of Chapter 1 on pages 1-9 (especially the account on homeomorphism) and Chapter 2 on pages 20-39 provide important background information for this session. Since the text on the Möbius strip and Klein bottle introduce experiments that may help in understanding the even more subtle projective plane, briefly skimming Chapters 3, 4, and 5 might be helpful.

The hope and intention for this session is to build our intutions about topology and thinking topologically about surfaces through exploring paper models of the projective plane.

It is recommended that participants build as many of the paper models discussed in the book as they have time for. Building the models and thinking about them ahead of time will help with our task of exploring the subject during our discussion.

To guide your exploration of the text and to guide our exploration during the meetup, I have organized this list of 22 questions:
http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.06.pdf

Here are some selected questions for the discussion from the full list (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.06.pdf):

● Inspired by the book, the questions that follow or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

● According to chapter 1 of Barr's book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr's definition compare to Wikipedia's definition of homeomorphism?

● Why would a projective plane with a hole in it be deformable to a Möbius strip? Is the projective plane with a hole in it, therefore, homeomorphic to the Moebius strip?

● Why is a sphere with a hole in it deformable to a plane? Is a sphere with a hole in it, therefore, homeomorphic to a plane?

● What is a cross-cap? How would you describe it? How would you describe it to a child?

● On pages 82-85, Barr describes the Martin Gardner model of a projective plane (Figures 8-11). Do you understand the model? How does it work? How does your imagination interpret the model to see the inherent self-intersection that is required?

● In several of the experiments discussed in Chapter 6 on "The Projective Plane" subtleties associated with connectivity when cutting and re-joining models are explored. What are these subtleties? What is the topological point of exploring them in our paper models? What caveats must we keep in mind when interpreting a model with cuts in it?

● The point of the latter part of Chapter 6 is to determine if the projective plane (and the Möbius strip) is symmetric. What does it mean for a topological surface to be symmetrical? How can you explain the symmetry of the projective plane with (paper) models? Can you imagine an explanation that is clear enough to explain the symmetry of the projective plane to a child?

This meetup will be part of a series exploring the content of Stephen Barr's fun book "Experiments in Topology".

"Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)

"Experiments in Topology" at Google Books (https://books.google.com/books?id=9TMx6ABV-98C)

Each event will be made as accessible to newcomers as possible. Key concepts will be reviewed and an effort to explain technical terms will be made. If anything is unfamiliar to you, please ask and we will try to clarify.

Exploring the Topology of the Projective Plane

Hosted By
CJ F. and Sam B.

Details

Stephen Barr's fun book "Experiments in Topology" helps us begin to think topologically by building and considering paper models. In this session, we will explore the projective plane. In Barr's book the projective plane is introduced in Chapter 2 on pages 34 and 38 and more extensively in Chapter 6 on pages 78-107.

The introductory section of Chapter 1 on pages 1-9 (especially the account on homeomorphism) and Chapter 2 on pages 20-39 provide important background information for this session. Since the text on the Möbius strip and Klein bottle introduce experiments that may help in understanding the even more subtle projective plane, briefly skimming Chapters 3, 4, and 5 might be helpful.

The hope and intention for this session is to build our intutions about topology and thinking topologically about surfaces through exploring paper models of the projective plane.

It is recommended that participants build as many of the paper models discussed in the book as they have time for. Building the models and thinking about them ahead of time will help with our task of exploring the subject during our discussion.

To guide your exploration of the text and to guide our exploration during the meetup, I have organized this list of 22 questions:
http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.06.pdf

Here are some selected questions for the discussion from the full list (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.06.pdf):

● Inspired by the book, the questions that follow or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

● According to chapter 1 of Barr's book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr's definition compare to Wikipedia's definition of homeomorphism?

● Why would a projective plane with a hole in it be deformable to a Möbius strip? Is the projective plane with a hole in it, therefore, homeomorphic to the Moebius strip?

● Why is a sphere with a hole in it deformable to a plane? Is a sphere with a hole in it, therefore, homeomorphic to a plane?

● What is a cross-cap? How would you describe it? How would you describe it to a child?

● On pages 82-85, Barr describes the Martin Gardner model of a projective plane (Figures 8-11). Do you understand the model? How does it work? How does your imagination interpret the model to see the inherent self-intersection that is required?

● In several of the experiments discussed in Chapter 6 on "The Projective Plane" subtleties associated with connectivity when cutting and re-joining models are explored. What are these subtleties? What is the topological point of exploring them in our paper models? What caveats must we keep in mind when interpreting a model with cuts in it?

● The point of the latter part of Chapter 6 is to determine if the projective plane (and the Möbius strip) is symmetric. What does it mean for a topological surface to be symmetrical? How can you explain the symmetry of the projective plane with (paper) models? Can you imagine an explanation that is clear enough to explain the symmetry of the projective plane to a child?

This meetup will be part of a series exploring the content of Stephen Barr's fun book "Experiments in Topology".

"Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)

"Experiments in Topology" at Google Books (https://books.google.com/books?id=9TMx6ABV-98C)

Each event will be made as accessible to newcomers as possible. Key concepts will be reviewed and an effort to explain technical terms will be made. If anything is unfamiliar to you, please ask and we will try to clarify.

Discussion Questions and Problems for “Exploring the Topology of the Projective Plane”
(Revision: March 12, 20181)
On 24 March 2018 Math Counts will discuss “Exploring the Topology of the Projective Plane”2. The following questions are based on Stephen Barr’s fun book “Experiments in Topology”. Your attempts to address them will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page2.

1. Inspired by the book, the questions that follow, or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

2. According to chapter 1 of Barr’s book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr’s definition compare to Wikipedia’s definition of homeomorphism?

3. What is the problem with the point x in the picture of the self-intersection for a Klein bottle or a projective plane depicted in Figure 1 on page 79? Why is the point x “a little dubious”?

4. How is a projective plane with a hole in it deformable to a Mobius strip? Is the projective plane with a hole in it, therefore, homeomorphic to the Mobius strip?

5. How is a sphere with a hole in it deformable to a plane? Is a sphere with a hole in it, therefore, homeomorphic to a plane?

6. On page 79, Barr briefly describes a cross-cap. What is a cross-cap? How would you describe it? How would you describe it to a child?

7. On pages 79–82, Barr describes the deformation of a Mobius band to the connectivity diagram of a projective plane. Does this mean that the projective plane is homeomorphic to a Mobius band? Or is it the projective plane with a hole in it that is homeomorphic to a Mobius band? What is the point of this carefully described deformation? What are its implications?

8. The text suggests that the figure on the left of Figure 1 on page 79 represents a projective plane. Does it? Even if you interpret the “fake intersection” as it abstractly “should be”? How can you explain this figure as a projective plane? How would cutting the corner y off the figure give a cross-cap?

9. Why does Barr describe the end result of the deformation on pages 79–82 as “a sphere with one cross-cap”? What does that mean? Why is it “a somewhat distorted projective plane”? What is distorted about it?

10. On pages 82–85, Barr describes the Martin Gardner model of a projective plane (Figures 8–11). Do you understand the model? How does it work? How should your imagination interpret the model to “see” the inherent self-intersection that is required?

1Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found.
Compiled by CJ Fearnley. http://blog.CJFearnley.com.
2https://www.meetup.com/MathCounts/events/248142433/

11. On page 83, Barr asks “why does a M¨obius strip with only 1 twist give, when cut down the middle on its axis, a loop with 4 twists?” Does he mean 2 half-twists or 4 (does the explanation of a “twist” on page 82 clarify)? That seems to agree with the descriptions on pages 48 and 76, but the definition of a twist has changed? What is going on?

12. On page 85 in referring to the Gardner model in Figure 11, Barr asks “but the edge BA′ is half in front and half behind the edge AB′: is this allowable?” Can the required joints be made? If so, does it matter that the model is cut and distorted so each half goes separately to its join? Does that entail a discontinuity? Is it OK?

13. In Figures 12, 13, and 14 on pages 85–86, a Mobius band with a cut along half its length is experimented with. What is the purpose of the experiment? Can you duplicate the result of only a 2 twist cylinder? What is your interpretation of this result?

14. In Figure 15 on page 86 another experiment with slits is attempted. What are your results in duplicating the experiment? Do you also find a cylinder with no twists? What are the implications of these experiments?
15. In Figure 20 on page 88 and the associated text, a variation of the Gardner model is described. What is different about this model? Does it fix the problem with the cut flaps going to opposite sides? What are the results of the experiment? What does it mean?

16. In Figure 23 on page 90 and the associated text, a flat disk model of the projective plane is described. How can we interpret this as a projective plane? What are the implications of the dissection along aa′?
17. In figures 26 and 27 on page 93 and in the associated text, a set of experiments exploring the effects of cuts in cruciform models of the projective plane is described. What are the results?

18. In figure 28 on page 94 and the associated text, a “boned” version of the Gardner model is described. How should we interpret the cuts in the Gardner model when they are removed completely? What is the result of this experiment? What does it mean? What does it imply?

19. In Figure 29 on page 95 and the associated text, a 2 piece model of a Mobius strip is described. What is the effect of axial cuts along the strip? Can you now state the rule relating bisection of these surfaces to the number of twists in the result?

20. In several of the experiments discussed in Chapter 6 on “The Projective Plane” subtleties associated with connectivity when cutting and re-joining models are explored. What are these subtleties? What is the topological point of exploring them in our paper models? What caveats must we keep in mind when interpreting a model with cuts in it?

21. The point of the latter part of Chapter 6 is to determine that the projective plane and the Mobius strip are symmetric. What does it mean for a topological surface to be symmetrical? Are you convinced they are symmetrical? How could you explain this symmetry with (paper) models? Could you explain it clearly enough to convince a child that both the M¨obius strip and the projective plane are symmetrical?

22. How are the topological and geometrical projective planes related? Can you see both in the models at http://blog.cjfearnley.com/2012/07/24/models-of-projective-geometry?

Map Coloring; Martin Gardner's Projective Plane & variations

Details

We will explore two topics in Stephen Barr's book "Experiments in Topology": 1) Map coloring (Chapter 7), and 2) Martin Gardner's projective plane and some variations on it (pages 82-91 of Chapter 6). Most of our time will be spent exploring the Chapter 6 material on the projective plane.

The introductory section of Chapter 1 on pages 1-9 (especially the account on homeomorphism) and Chapter 2 on pages 20-39 provide important background information for the Chapter 6 material.

To guide your exploration of the text and to guide our exploration during the meetup, I have organized this list of 11 questions:
http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.07.pdf

Three of the questions are on map coloring. The rest of the questions (8) explore detailed experiments about the Gardner model of the projective plane and some variants. The aim of the models is to provide some insights into the symmetry of the projective plane. You will need the questions and the book in order to look review them before the meetup. Building the models is recommended as it will allow us to explore the subject in much more depth and breadth.

This meetup is part of a series exploring the content of Stephen Barr's fun book "Experiments in Topology".

"Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)

"Experiments in Topology" at Google Books (https://books.google.com/books?id=9TMx6ABV-98C)

Each event will be made as accessible to newcomers as possible. Key concepts will be reviewed and any technical terms will be explained. If anything is unfamiliar to you, please ask and we will try to clarify.

[CJ F.](#)

2,622 days ago

I really valued Kurt's observation that the basic argument of Chapter 7 on Map Coloring is roughly the idea behind the computer-generated proofs of the four-color theorem: identify some 2000 cases of regions (as Barr describes them) and program a computer to check each case to ensure that 4 colors suffices. It made me appreciate the chapter much more.

Stephen Barr has certainly proven to be an able guide in our quest to apprehend something of topology. Every unclear part of the book has proven, after some doubts and a deep and long discussion, to be eloquently worded to hint at a Big idea without the usual accoutrements of complex notions and the graduate level study needed to understand them. That is a gift. It makes the book reference quality for my bookshelf.

I have a draft event description for May 26th and PDF with questions posted. I will probably edit both further before RSVPs open on May 12th, so don't print them yet. But let me know if anything is amiss.

[Greg G.](#)

2,626 days ago

C J Sorry something came up that I can’t make the meeting today looking forward to next month Greg

[CJ F.](#)

2,626 days ago

Tomorrow we will explore two topics in Stephen Barr's book "Experiments in Topology": Map Coloring & Martin Gardner's model of the Projective Plane (and some variations on it).

To guide our exploration, I have prepared 11 questions:
http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.07.pdf

My previous comment gave links to Google Books to read the parts of the book relevant to our topic (Chapter 7 on Map Coloring pp.108-119; Chapter 6 on the Projective Plane pp.82-91).

For our newcomers, you can get a lot out of our events even if you do not prepare. So join us even if you haven't prepared.

Several of us, take the opportunity to dig in deep to find the blocks to our understanding. Mathematics is doing and only by doing can we improve our skills and understanding. For us, the events help us overcome hurdles in our mathematics and preparation is the way we make our events richer.

Do you have any questions on the 11 prepared questions or the reading in the book?

CJ F.

2,626 days ago

The Map Coloring chapter has some great stuff in it but also some unclear passages. My feeling is it doesn't give enough clues to understand it deeply (I feel a better book might help), so I only included 3 questions on coloring. If your questions concern one of the adequately explained parts of the chapter, it will be worthwhile discussing at length. Otherwise, I feel it might bog us down too much and I might judiciously suggest (if no one can offer any insights in a reasonable amount of time) that we defer some questions until we find a better resource.

I'm really enjoying the the Projective Plane material and I feel the book and our effort will be rewarded by a deep dive into every nuanced clause in the book. So I look forward to get bogged down in it and to looking at every comment and question from all sides until we're too exhausted to continue!

Good luck preparing and let us know if you have any questions or thoughts about the questions or the book.

+3

Map Coloring; Martin Gardner's Projective Plane & variations

Map Coloring; Martin Gardner's Projective Plane & variations

Discussion Questions and Problems for “Map Coloring; Martin
Gardner’s Projective Plane & variations”
(Revision: April 23, 20181)
On 28 April 2018 Math Counts will discuss “Map Coloring; Martin Gardner’s Projective Plane & variations”2. The following questions are based on Stephen Barr’s fun book “Experiments in Topology”. Your attempts to address them will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page2.

1. Inspired by the book, the questions that follow, or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

2. In Figure 1 on page 111, there is a map coloring of the torus that requires seven colors. Show that that map cannot be colored with six colors where a valid coloring requires that no two

regions of the same color share a common segment of an edge.

3. In Figure 2 on page 111, there is a map coloring of the Mobius band that requires six colors. Show that that map cannot be colored with five colors.

4. In the PUZZLE on pages 118 and 119, color the map in Figure 17 subject to the constraint that you only have 24 square feet of RED, 24 square feet of YELLOW, and 16 square feet of GREEN.

5. On pages 82–85, Barr describes the Martin Gardner model of a projective plane (Figures 8–11). Do you understand the model? How does it work? How should your imagination interpret the model to "see" the inherent self-intersection that is required?

6. On page 83, Barr asks "why does a Mobius strip with only 1 twist give, when cut down the middle on its axis, a loop with 4 twists?" Does he mean 2 half-twists or 4 (does the explanation of a "twist" on page 82 clarify)? That seems to agree with the descriptions on pages 48 and 76, but the definition of a twist has changed? How can we count the number of twists in a topological surface? What is going on?

7. On page 85 in referring to the Gardner model in Figure 11, Barr asks "but the edge BA′ is half in front and half behind the edge AB′: is this allowable?" Can the required joints be made? If so, does it matter that the model is cut and distorted so each half goes separately to its join? Does that entail a discontinuity? Is there some disconnectedness? Is it OK?

8. In Figures 12 and 15 on pages 85–86, a M¨obius band with cuts along half the centerline of both its length and width is experimented with. How are these cuts related to the Gardner model of the projective plane? What is the purpose of the experiments? What are the results of the experiments? Can you duplicate the results? Did you get both a two twist and a no twist cylinder? What is your interpretation of these results? What are the implications? Is it OK to cut slits in our models so long as we re-attach them with the correct connectivity as suggested by the definition of homeomorphism given in Chapter 1 of the book? What pitfalls must we be attentive to?

9. In Figure 19 on page 87 and in Figure 20 on page 88 and the associated text, two variations of the Gardner model are described. What is different about these models? Do they fix the problem with the cut flaps going to opposite sides? What are the results of the experiments? Do you also get a loop with two twists? Why is that the result? What does it mean?

10. In Figure 23 on page 90 and the associated text, a flat disk model of the projective plane is described. How can we interpret this as a projective plane? What are the implications of the dissection along aa′? Can every projective plane be so dissected? Why? Does that suggest that the projective plane is asymmetrical?

11. Do the results of these experiments suggest that the projective place is a symmetrical topological surface? Why? Why not?

1Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found.
Compiled by CJ Fearnley. http://blog.CJFearnley.com.
2https://www.meetup.com/MathCounts/events/249730982/

The Symmetry of the Projective Plane (and the curious property of twist)

Hosted By
CJ F. and Sam B.

Details

We will explore a number of models of the projective plane with a view toward understanding its symmetry. Our trusty guide will be pages 82-107 of Chapter 6 in Stephen Barr's "Experiments in Topology". There is an extensive discussion on twist in the chapter which we will want to understand. If you are new to the book reading pages 1-9 in Chapter 1 and all of Chapter 2 (pages 20-39) will provide sufficient background.

Here is a set of 15 exercises which will guide our exploration during the event:
http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.08.pdf

Here are the main questions for the discussion abstracted from the full list of exercises (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.08.pdf):

● How does the Martin Gardner model of the projective plane work?

● What is the pattern for the number of twists that result after cutting along the axes of various variations on the Martin Gardner model of the projective plane discussed in Stephen Barr's text?

● In considering this rule for the number of twists and the full set of variations on the Martin Gardner model of the projective plane discussed in Stephen Barr's text, can you understand why the projective plane is symmetrical? Can you organize your understanding to be clear enough to explain the projective plane's symmetry to a child?

● How are the topological and geometrical projective planes related? Can you see the symmetry of the topological perspective in each of the geometrical models described at http://blog.cjfearnley.com/2012/07/24/models-of-projective-geometry

"Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)

"Experiments in Topology" at Google Books (https://books.google.com/books?id=9TMx6ABV-98C)

The event will be made as accessible to newcomers as possible. Key concepts will be reviewed and an effort to explain technical terms will be made. If anything is unfamiliar to you, please ask and we will try to clarify.

Discussion Questions for “The Symmetry of the Projective Plane (and
the curious property of twist)”
(Revision: May 17, 20181)
On 26 May 2018 Math Counts will discuss “The Symmetry of the Projective Plane (and the curious

property of twist)”[2]. The following questions are based on Stephen Barr’s fun book “Experiments in Topology”. Your attempts to address them will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page[2].

1. On pages 82–85, Barr describes the Martin Gardner model of a projective plane (Figures 8–11). Do you understand the model? How does it work?

2. On page 85 in referring to the Gardner model in Figure 11, Barr asks “but the edge BA′ is half in front and half behind the edge AB′: is this allowable?” Does that entail a discontinuity? Is there some disconnectedness? Is it OK?

3. On page 83, Barr asks “why does a Mobius strip with only 1 twist give, when cut down the middle on its axis, a loop with 4 twists?” Does he mean 2 half-twists or 4 (does the explanation of a “twist” on page 82 clarify)? That seems to agree with the descriptions on pages 48 and 76, but the definition of a twist has changed? How can we count the number of twists in a topological surface? What is going on?

4. In Figures 12 and 15 on pages 85–86, two oblong Gardner models where we do not glue one pair of edges together are explored. Because only one pair of opposite sides are glued, it is a Mobius strip. Can you duplicate the results of the experiments? Did you get both a two twist and a no twist cylinder? What is your interpretation of these results? What are the implications? Is it OK to cut slits in our models so long as we re-attach them with the correct connectivity as suggested by the definition of homeomorphism given on pages 4–5 in Chapter 1 of the book? Why are the number of twists different in the two cases? Is twist a topological property? Is it part of homeomorphism or geometry?

5. In Figure 19 on page 87 and in Figure 20 on page 88 and the associated text, two variations of the Gardner model are described. Do these models fix the problem with the cut flaps going to opposite sides (see question #2)? Do you also get a loop with two twists when you cut these models as in Figures 13 and 15 on pages 85 and 86 (see question #4)? Why is that? What does it mean?

6. In the circular form of Gardner’s model, Figure 23 on page 90 and the associated text, a flat disk model of the projective plane is described. How can we interpret this as a projective plane? What are the implications of the dissection along aa′? Can every projective plane be so dissected? Why? Why not? Does that suggest that the projective plane is asymmetrical?

7. In Figures 26 and 27 on page 93 and in the associated text, a set of experiments exploring the effects of axial cuts in cruciform models of the projective plane is described. What are the results and implications?

[1] Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found.
Compiled by CJ Fearnley. http://blog.CJFearnley.com.
[2] https://www.meetup.com/MathCounts/events/250256203/

8. In Figure 28 on page 94 and the associated text, a “boned” version of the Gardner model is described. How should we interpret the cuts in the Gardner model when they are removed completely? What are the results and implications of this experiment?

9. In Figure 29 on page 95 and the associated text, a two-piece model of a Mobius strip is described. What is the effect of axial cuts along the strip? Can you now state the rule relating axial bisections of these surfaces to the number of twists in the result?

10. Pages 96–102 describe the experiment of giving paper strips various numbers of twists, then glueing their ends together, then cutting along their axes, and then counting the number of twists in the result. Can you duplicate the experiment and its results? Why does “increasing an odd number of twists by one fail to increase the final number after cutting”? Why does increasing an even number of twists by one add “4 new twists” after cutting? Why do all the strips with 2n + 1 twists for n a counting number, give, after cutting, a loop with a knot in it? How does this analysis apply to axially cutting the Mobius band as explored in question #3, the oblong Gardner models in #4, the flat Mobius stip in #9, the cruciform models in #7, and the boned version of the Gardner model in #8? How could you explain this twisting rule to a child so they might understand it?

11. On pages 102–105, the circular form of the Gardner model (see question #6) is re-examined. Do you understand the cut in that model that gives the symmetrical Mobius strip of Figure 13 on page 85 (see question #4)? Do you understand the implications of cutting through the center of this model (point C in Figure 23 on page 90)? Do you see how to cut a right- and left-handed M¨obius strip out of this model by including more in the cross-cap piece than what would be given by a straight cut through the center? Do you see how to cut the cruciform models of the projective plane out of this model (see Figure 26 and 27 on page 93 and question #7)? Do these considerations “prove” that the projective plane is really symmetrical?

12. On pages 101–102, and again on 105–106, the boned Gardner model is re-examined (see Figure 28 on page 94 and question #8). In Figure 44 on page 106, several “bonings” of other variations of the Gardner model are considered. How do these models support the case that the projective plane is symmetrical?

13. The conclusion of Chapter 6 is that the projective plane and the Mobius strip are symmetrical. What does it mean for a topological surface to be symmetrical? Are you convinced? How could you explain this symmetry clearly enough to convince a child that both the Mobius strip and the projective plane are symmetrical?

14. Does the sequence of experiments with various variations of Martin Gardner’s model of the projective plane reveal how imperfect and “lowly” (as Barr calls them on page 106) paper models can help one identify important abstract topological properties? Can geometrical properties such as twist and embeddings in 3-space help the experimental topologist explore and more deeply understand topological surfaces? Does this effort demonstrate that mathematics could be (or maybe even is) an experimental science? Is abstract mathematics simply putting in order the results of a large number of examples?

15. How are the topological and geometrical projective planes related? Can you see the topological perspective in each of the geometrical models described at http://blog.cjfearnley.com/2012/07/24/models-of-projective-geometry?

Using Twist to Understand the Topology of the Projective Plane & its Symmetry

Hosted By
CJ F. and Sam B.

Details

We will explore the nature of twist in a number of paper models of the projective plane with a view toward understanding its symmetry and its topology. Our trusty guide will be pages 82-107 of Chapter 6 in Stephen Barr's "Experiments in Topology". If you are new to the book reading pages 1-9 in Chapter 1 and all of Chapter 2 (pages 20-39) will provide sufficient background.

Here is a set of 11 exercises which will guide our exploration during the event:
http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.09.pdf

Here are the main questions for the discussion abstracted from the full list of exercises (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.09.pdf):

● How does the Martin Gardner model of the projective plane work?

● What is the pattern for the number of twists that result after cutting along the axes of various variations on the Martin Gardner model of the projective plane discussed in Stephen Barr's text?

● In considering this rule for the number of twists and the full set of variations on the Martin Gardner model of the projective plane discussed in Stephen Barr's text, can you understand why the projective plane is symmetrical? Can you organize your understanding to be clear enough to explain the projective plane's symmetry to a child?

● How are the topological and geometrical projective planes related? Can you see the symmetry of the topological perspective in each of the geometrical models described at http://blog.cjfearnley.com/2012/07/24/models-of-projective-geometry

"Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)

"Experiments in Topology" at Google Books (https://books.google.com/books?id=9TMx6ABV-98C)

The event will be made as accessible to newcomers as possible. Key concepts will be reviewed and an effort to explain technical terms will be made. If anything is unfamiliar to you, please ask and we will try to clarify.

Discussion Questions for “Using Twist to Understand the Topology of
the Projective Plane & its Symmetry”
(Revision: June 10, 20181)
On 23 June 2018 Math Counts will discuss “Using Twist to Understand the Topology of the Projective Plane & its Symmetry”2. The following questions are based on Stephen Barr’s fun book “Experiments in Topology”. Your attempts to address them will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page2.

1. On pages 82–85, Barr describes the Martin Gardner model of a projective plane (Figures 8–11). Do you understand the model? How does it work?

2. In the circular form of Gardner’s model, Figure 23 on page 90 and the associated text, a flat disk model of the projective plane is described. How can we interpret this as a projective plane? What are the implications of the dissection along aa′? Can every projective plane be so dissected? Why? Why not? Does that suggest that the projective plane is asymmetrical?

3. In Figures 26 and 27 on page 93 and in the associated text, a set of experiments exploring the effects of axial cuts in cruciform models of the projective plane is described. What are the results and implications?

4. In Figure 28 on page 94 and the associated text, a “boned” version of the Gardner model is described. How should we interpret the cuts in the Gardner model when they are removed completely? What are the results and implications of this experiment?

5. In Figure 29 on page 95 and the associated text, a two-piece model of a Mobius strip is described. What is the effect of axial cuts along the strip? Can you now state the rule relating axial bisections of these surfaces to the number of twists in the result?

6. Pages 96–102 describe the experiment of giving paper strips various numbers of twists, then glueing their ends together, then cutting along their axes, and then counting the number of twists in the result. Can you duplicate the experiment and its results? Why does “increasing an odd number of twists by one fail to increase the final number after cutting”? Why does increasing an even number of twists by one add “4 new twists” after cutting? Why do all the strips with 2n + 1 twists for n a counting number, give, after cutting, a loop with a knot in it? How does this analysis apply to axially cutting the M¨obius band as explored on pages 83, 48 and 76, the oblong Gardner models in Figures 12, 13, and 15 on pages 85–86, the flat M¨obius strip in #5, the cruciform models in #3, and the boned version of the Gardner model in #4? How could you explain this twisting rule to a child so they might understand it?

[1]Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found.
Compiled by CJ Fearnley. http://blog.CJFearnley.com.
[2]https://www.meetup.com/MathCounts/events/251163336/

7. On pages 102–105, the circular form of the Gardner model (see question #2) is re-examined. Do you understand the cut in that model that gives the symmetrical Mobius strip of Figure 13 on page 85? Do you understand the implications of cutting through the center of this model (point C in Figure 23 on page 90)? Do you see how to cut a right- and left-handed Mobius strip out of this model by including more in the cross-cap piece than what would be given by a straight cut through the center? Do you see how to cut the cruciform models of the projective plane out of this model (see Figure 26 and 27 on page 93 and question #3)? Do these considerations “prove” that the projective plane is really symmetrical?

8. On pages 101–102, and again on 105–106, the boned Gardner model is re-examined (see Figure 28 on page 94 and question #4). In Figure 44 on page 106, several “bonings” of other variations of the Gardner model are considered. How do these models support the case that the projective plane is symmetrical?

9. The conclusion of Chapter 6 is that the projective plane and the Mobius strip are symmetrical. What does it mean for a topological surface to be symmetrical? Are you convinced? How could you explain this symmetry clearly enough to convince a child that both the Mobius strip and the projective plane are symmetrical?

10. Does the sequence of experiments with various variations of Martin Gardner’s model of the projective plane reveal how imperfect and “lowly” (as Barr calls them on page 106) paper models can help one identify important abstract topological properties? Can geometrical properties such as twist and embeddings in 3-space help the experimental topologist explore and more deeply understand topological surfaces? Does this effort demonstrate that mathematics could be (or maybe even is) an experimental science? Is abstract mathematics simply

putting in order the results of a large number of examples?

11. How are the topological and geometrical projective planes related? Can you see the topological perspective in each of the geometrical models described at http://blog.cjfearnley.com/2012/07/24/models-of-projective-geometry?

Betti Numbers and the Symmetry of the Projective Plane

Hosted By
CJ F. and Sam B.

Details

We will explore several questions on Betti numbers and the nature and the symmetry of the projective plane from chapters 8 and 6 of Stephen Barr's book "Experiments in Topology". We will focus on pages 123-128 in Chapter 8 which discusses Betti numbers and pages 96-107 in Chapter 6 on the projective plane.

All participants might want to read pages 10-19 in Chapter 1 on Euler's theorem which is referenced in the material on Betti numbers. For completeness, you might want read all of Chapter 8, pages 120-135, as well.

Newcomers to the subject are advised to read pages 1-9 in Chapter 1, all of Chapter 2 (pages 20-39), and the beginning of Chapter 6 (pages 82-96) in order to follow the material on the projective plane.

Here is a set of 10 exercises which will guide our exploration during the event: http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.10.pdf

Here are the main questions for the discussion abstracted from the full list of exercises (http://www.cjfearnley.com/MathCounts/TopologicalSurfaces.10.pdf):

● What is the Betti number for a disk, an annulus, a torus, a Möbius strip, a Klein bottle, a sphere, and a projective plane? Do the different results reasonably characterize the connectivity of each surface? How? Why?

● What is the pattern for the number of twists that result after cutting along the axes of various variations on the Martin Gardner model of the projective plane discussed in Stephen Barr's text?

● In considering this rule for the number of twists and the full set of variations on the Martin Gardner model of the projective plane discussed in Stephen Barr's text, can you understand why the projective plane is symmetrical? Can you organize your understanding to be clear enough to explain the projective plane's symmetry to a child?

● How are the topological and geometrical projective planes related? Can you see the symmetry of the topological perspective in each of the geometrical models described at http://blog.cjfearnley.com/2012/07/24/models-of-projective-geometry

This meetup is part of a series exploring the content of Stephen Barr's fun book "Experiments in Topology".

"Experiments in Topology" is available from Dover (http://store.doverpublications.com/0486259331.html)

"Experiments in Topology" at Google Books (https://books.google.com/books?id=9TMx6ABV-98C)

Each event will be made as accessible to newcomers as possible. Key concepts will be reviewed and an effort to explain technical terms will be made. If anything is unfamiliar to you, please ask and we will try to clarify.

Discussion Questions for “Betti Numbers and the Symmetry of the Projective Plane”
(Revision: July 18, 20181)
On 28 July 2018 Math Counts will discuss “Betti Numbers and the Symmetry of the Projective Plane”2. The following questions are based on Stephen Barr’s fun book “Experiments in Topology”.
Your attempts to address them will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page2.

1. On pages 123, 124, and 126, Barr provides three definitions for Betti numbers. The first based on the number of edges that need to be removed from a network or graph to produce a connected tree. The second based on cross-cuts and the third based on loop-cuts. Are all three definitions topologically equivalent? Why? Why not?

2. How (in what ways) do the three definitions of Betti numbers characterize the connectivity of a topological surface?

3. What is the Betti number for a disk, an annulus, a torus, a M¨obius strip, a Klein bottle, a sphere, and a projective plane? Do the different results reasonably characterize the connectivity of each surface? How? Why?

4. Pages 96–102 describe the experiment of giving paper strips various numbers of twists, then glueing their ends together, then cutting along their axes, and then counting the number of twists in the result. Can you duplicate the experiment and its results? Why does “increasing an odd number of twists by one fail to increase the final number after cutting”? Why does increasing an even number of twists by one add “4 new twists” after cutting? Why do all the strips with 2n + 1 twists for n a counting number, give, after cutting, a loop with a knot in it? How could you explain this twisting rule to a child so they might understand it?

5. How does this analysis apply to axially cutting the M¨obius band as explored on pages 83, 48 and 76, the oblong Gardner models in Figures 12, 13, and 15 on pages 85–86, the flat M¨obius strip (Figure 29 on page 95), the cruciform models (Figures 26 and 27 on page 93), and the boned version of the Gardner model (Figure 28 on page 94)?

6. On pages 102–105, the circular form of the Gardner model (see Figure 23 on page 90) is re-examined. Do you understand the cut in that model that gives the symmetrical M¨obius

strip (see Figures 13 and 14 on pages 85–86)? Do you understand the implications of cutting through the center of this model (point C in Figure 23 on page 90)? Do you see how to cut a right- and left-handed M¨obius strip out of this model by including more in the cross-cap piece than what would be given by a straight cut through the center? Do you see how to cut the cruciform models of the projective plane out of this model (see Figures 26 and 27 on page 93)? Do these considerations “prove” that the projective plane is really symmetrical?

7. On pages 101–102, and again on 105–106, the boned Gardner model is re-examined (see Figure 28 on page 94). In Figure 44 on page 106, several “bonings” of other variations of the Gardner model are considered. How do these models support the case that the projective plane is symmetrical?
1Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found.
Compiled by CJ Fearnley. http://blog.CJFearnley.com.
2https://www.meetup.com/MathCounts/events/252093681/

8. The conclusion of Chapter 6 is that the projective plane and the Mobius strip are symmetrical. What does it mean for a topological surface to be symmetrical? Are you convinced? How could you explain this symmetry clearly enough to convince a child that both the Mobius strip and the projective plane are symmetrical?

9. Does the sequence of experiments with various variations of Martin Gardner’s model of the projective plane reveal how imperfect and “lowly” (as Barr calls them on page 106) paper models can help one identify important abstract topological properties? Can geometrical properties such as twist and embeddings in 3-space help the experimental topologist explore and more deeply understand topological surfaces? Does this effort demonstrate that mathematics could be (or maybe even is) an experimental science? Is abstract mathematics simply putting in order the results of a large number of examples?

10. How are the topological and geometrical projective planes related? Can you see the topological perspective in each of the geometrical models described at
http://blog.cjfearnley.com/2012/07/24/models-of-projective-geometry?

Deliberations in The Trial of the Punctured Torus

Hosted By
CJ F. and Sam B.

Details

Can a torus (the surface of an inner tube or a doughnut) be turned inside-out? Is there one way to do it or more than one way? We will explore the merits and demerits of two possible ways to turn a torus inside-out presented by Mr. Jones and Dr. Situs in "The Trial of the Punctured Torus" which is Chapter 9 in Stephen Barr's fun book "Experiments in Topology".

If you are new to the book reading pages 1-9 in Chapter 1, pages 20-34 in Chapter 2, and all of Chapter 9 (pages 136-148) will provide sufficient background.

Here are the questions that will guide our exploration of the subject:

- Do you understand Dr. Situs' procedure for turning the punctured torus inside out? Can you explain it? Can you demonstrate it?
- Explain the resemblance (or lack thereof) between the statue of Laocoön and His Sons (see https://en.wikipedia.org/wiki/Laoco%C3%B6n_and_His_Sons) and the drawings in Figures 1-9 on pages 136-139.
- Do you agree with Mr. Jones that Dr. Situs cannot complete the inside-outing process beyond Figure 9 on page 139? Do you agree with Jones that Situs' torus is not really inside-out?

- Is Dr. Situs' allegedly inside-out torus homeomorphically equivalent to a torus? Or is it something else? What?

- What is the point of Mr. Jones' demonstration of turning a glove inside-out (see Figures 15-19 on pages 141-142)? If even one finger is not turned inside-out is the whole, therefore, not inverted? Is the partly inside-out glove homeomorphic to a right-handed glove?

- Do you agree with Mr. Jones that the two linked closed curves (the dotted and dashed lines in Figures 3-13 on pages 138-140) stay linked at the end of Dr. Situs' inversion of the punctured torus? Is Mr. Jones correct that "two closed curves that are linked cannot be unlinked by any topological [homeomorphic] deformation"? If true, does that prove that Dr. Situs' inversion is inadequate?

- In Figures 22-26 on page 144, Mr. Jones describes his procedure for inside-outing a torus. Is his cut of the torus an admissible homeomorphic deformation because he glues it back together with the same connectedness it had before he cut it? Is Mr. Jones' inversion valid? Is his approach better or worse than Dr. Situs'? Why?

- Do the linked dotted and dashed closed curves in Figures 22-26 on page 144 show that Mr. Jones' inversion is valid and Dr. Situs' is invalid? Is linkedness a topological invariant?

- Do both Dr. Situs' and Mr. Jones' inversions turn surface fur from the outside to the inside and vice versa as depicted in Figures 27-30 on page 146? Does that prove that both inversions are valid?

- Do both Dr. Situs' and Mr. Jones' inversions turn the grain of the surfaces from cylindrical to annular as in Figures 31-32 on page 147? Is this grain transformation a requirement for inside-outing?

- The deliberations in this trial consider inversions of a torus that 1) change the inner surface to the outer surface, 2) change the "grain" from cylindrical to annular, 3) preserve or break linked Jordan curves. Are any of these criteria requirements for inverting a torus? Which ones? Why?

● Do you consider Dr. Situs' or Mr. Jones' inversion to be valid? Which, if any, is invalid? Why?

● What are the topological requirements for inside-outing? How would you define topological inversion of surfaces?

This is the last topic in a series exploring Stephen Barr's fun book "Experiments in Topology".

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